On Solving Transportation Problem in Fuzzy Environment Using Ranking Function

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Abstract

Transportation problem is a very important problem that has been wide studied in operations research domain. It has been usually accustomed simulate completely different world issues. The main aspect of this paper is to search out the smallest amount transportation value of some commodities through a capacitated network once the availability and demand of nodes and therefore the capacity and value of edges area unit represented as fuzzy numbers. we are solving the transportation problem using the Robust ranking technique, where fuzzy demand and supply all are in the form of trapezoidal fuzzy numbers. The fuzzification of the price of the transportation problem is mentioned with the assistance of a numerical example.

Key words: *Transportation problem, Fuzzy Transportation problem, Trapezoidal fuzzy numbers, Robust Ranking Technique.*

1. Introduction

The transportation problem is one amongst the earliest applications of applied mathematics issues. Transportation models have wide applications in supply and provide chain for reducing the value economical algorithms are developed for determination the transportation problem once the value coefficients and the supply and demand quantities are known exactly. The prevalence of randomness and inexactitude within the world is inevitable as a result of some unexpected situations. There square measure cases that the value coefficients and also the offer and demand quantities of a transportation problem could also be unsure due to some uncontrollable factors. To deal quantitatively with imprecise information in making decisions Bellman et.al.,[2] introduced the notion of fuzziness.

Let a_i be the number of units of a product available at origin i and b_j be the number of units of the product required at destination j. Let C_{ij} be the cost of transporting one unit from origin i to destination j and let X_{ij} be the amount of quantity carried or shipped from origin i to destination j. There area unit effective algorithms for resolution the transportation issues once all the choice parameters, i.e. the supply obtainable at every supply, the demand needed at every destination also because the unit transportation prices area unit given during a precise approach. But in world, there area unit several numerous things due to uncertainty in one or additional call parameters and thence they will not be expressed during a precise approach. This is due to measurement quality, lack of proof, computational errors, high information cost, whether conditions etc. Hence, we tend to cannot apply the normal classical ways to resolve the transportation issues with success. Let be the number of units of a product available at origin and be the number of units of the product required at destination. Let be the cost of transporting one unit from origin to destination and let be the amount of quantity carried or shipped from origin to destination.

Therefore the utilization of Fuzzy transportation problems is additional applicable to model and solve the important world problems. A fuzzy transportation problem may be a transportation problem within which the transportation prices, provide and demand area unit fuzzy quantities.

Bellman and Zadeh [3] projected the idea of higher cognitive process in Fuzzy surroundings. After this pioneering work, many authors like Shiang-Tai Liu and Chiang Kao[16], Chanas et al[5], Pandian et.al [14], Liu and Kao [11] etc proposed different methods for the solution of Fuzzy transportation problems.

Chanas and Kuchta [4] projected the idea of the optimum answer for the Transportation with Fuzzy constant expressed as Fuzzy numbers. Chanas, Kolodziejckzy, Machaj[5] presented a Fuzzy linear programming model for solving Transportation problem.

Liu and Kao [11] represented a way to resolve a Fuzzy Transportation downside supported extension principle.

Lin introduced a genetic algorithmic program to resolve Transportation with Fuzzy objective functions. Srinivasan [18] - [23] described the new methods to solve fuzzy transportation problem. Pandian and Natarajan [14] proposed a Fuzzy zero point method for finding a Fuzzy optimal solution for Fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers.

2. PRELIMINARIES

In this section, we define some basic definitions, which will be used in this paper.

2.1 Definition – 1

The characteristic function $\mu_A(x)$ of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}(x)$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned value indicate the membership grade of the element in the set A. The function $\mu_{\tilde{A}}(x)$ is called the membership function and the set $\tilde{A} = \{(x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0,1]\}$. is called a fuzzy set.

2.2 Definition – 2

A fuzzy set A, defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_A : R \to [0,1]$ has the following characteristics

(i) A is normal. It means that there exists an x ∈ R such that $\mu_A(x) = 1$

(ii) A is convex. It means that for every $x_1, x_2 \in R$, $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\},\ \lambda \in [0,1]$ (iii) μ_A is upper semi-continuous.

(iv) supp (A) is bounded in R.

2.3 Definition – 3

A fuzzy number A is said to be non-negative fuzzy number if and only $\mu_{\tilde{A}}(x) = 0,\ \forall\, x < 0$

2.4 Definition – 4

A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by, where a≤ b≤ c≤d.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a < x \leq b, \\ 1, & b < x < c, \\ \dfrac{d-x}{d-c}, & c \leq x < d, \\ 0, & x > d \end{cases}$$

2.5 Definition – 5

A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be non-negative (non positive) trapezoidal fuzzy number. i.e. $A \geq 0 (A \leq 0)$ if and only if $a \geq 0 (c \leq 0)$. A trapezoidal fuzzy number is said to be positive (negative) trapezoidal fuzzy number i.e. $A > 0 (A < 0)$ if and only if $a > 0 (c < 0)$.

2.6 Definition – 6

Two trapezoidal fuzzy number $\tilde{A}_1 = (a, b, c, d)$ and $\tilde{A}_2 = (e, f, g, h)$ are said to be equal. i.e. $\tilde{A}_1 = \tilde{A}_2$ if and only if a=e, b=f, c=g, d=h.

2.7 Definition – 7

Let $\tilde{A}_1 = (a,b,c,d)$ and $\tilde{A}_2 = (e,f,g,h)$ be two non-negative trapezoidal fuzzy number then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (a,b,c,d) \oplus (e,f,g,h) = (a+e, b+f, c+g, d+h)$

(ii) $\tilde{A}_1 - \tilde{A}_2 = (a,b,c,d) - (e,f,g,h) = (a-h, b-g, c-f, d-e)$

(iii) $-\tilde{A}_1 = -(a,b,c,d) = (-d,-c,-b,-a)$

(iv) $\tilde{A}_1 \otimes \tilde{A}_2 = (a,b,c,d) \otimes (e,f,g,h) = (ae, bf, cg, dh)$

(v) $\frac{1}{\tilde{A}} \cong \left(1/d, 1/c, 1/b, 1/a \right)$

2.8 Robust Ranking Technique

Roubast ranking technique which satisfy compensation, linearity, and additivity properties and provides results which are consist human intuition. If ã is a fuzzy number then the Roubast Ranking is defined by $R(\tilde{a}) = \int_0^1 0.5(a_\alpha^L a_\alpha^U) d\alpha$, Where $(a_\alpha^L a_\alpha^U)$ is the α level cut of the fuzzy number $\tilde{a}$ and $(a_\alpha^L a_\alpha^U) = \{((b-a)+a),(d-(d-c))\}$

In this paper we use this method for ranking the objective values. The Roubast ranking index $R(\tilde{a})$ gives the representative value of fuzzy number $\tilde{a}$.

3. Mathematical formulation of a fuzzy transportation problem

Mathematically a transportation problem can be stated as follows:

Minimize

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \qquad \text{------(1)}$$

Subject to

$$\left.\begin{array}{ll} \sum_{j=1}^{n} x_{ij} = a_i & j = 1,2,\dots,n \\ \sum_{i=1}^{m} x_{ij} = b_j & i = 1,2,\dots,m \\ x_{ij} \ge 0 & i = 1,2,\dots,m,\ j = 1,2,\dots,n \end{array}\right\} \text{----(2)}$$

Where c_{ij} is the cost of transportation of an unit from the i^{th} source to the j^{th} destination, and the quantity x_{ij} is to be some positive integer or zero, which is to be transported from the i^{th} origin to j^{th} destination. A obvious necessary and sufficient condition for the linear programming problem given in (1) to have a solution is that

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j \qquad \text{------(3)}$$

(i.e) assume that total available is equal to the total required. If it is not true, a fictitious source or destination can be added. It should be noted that the problem has feasible solution if and only if the condition (2) satisfied. Now, the problem is to determine x_{ij}, in such a way that the total transportation cost is minimum

Mathematically a fuzzy transportation problem can be stated as follows:

Minimize

$$z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \qquad \text{------(4)}$$

Subject to

$$\left.\begin{array}{lll} \sum_{j=1}^{n} x_{ij} = \tilde{a}_i & j = 1,2,\ldots,n & \\ \sum_{i=1}^{m} x_{ij} = b_j & i = 1,2,\ldots,m & \\ x_{ij} \geq 0 & i = 1,2,\ldots,m, & j = 1,2,\ldots,n \end{array}\right\} \text{----(5)}$$

In which the transportation costs $\tilde{c}_{ij}$, supply a_i and demand $\tilde{b}_j$ quantities are fuzzy quantities. An obvious necessary and sufficient condition for the fuzzy linear programming problem give in (4-5) to have a solution is that

$$\sum_{i=1}^{n} a_i \;\square\; \sum_{j=1}^{m} \tilde{b}_j \qquad \text{------(6)}$$

This problem can also be represented as follows:

	1	……..	n	Supply
1	$\tilde{c}_{11}$	……..	$\tilde{c}_{1n}$	a_1
. . . .	. . . .	. . . .	. . . .	. . . .
m	$\tilde{c}_{m1}$	……..	$\tilde{c}_{mn}$	a_m
Demand	$\tilde{b}_1$	……..	$\tilde{b}_n$	

4. Method for Solving Transportation Problem

Following are the steps for solving Transportation Problem

Step – 1: From the given Transportation problem, convert fuzzy values to crisp values using ranking function.

Step – 2: Deduct the minimum cell cost from each of the cell cost of every row/column of the Transportation problem and place them on the right-top/right-bottom of corresponding cost.

Step – 3: Adding the cost of right-top and right – bottom and place the summation value in the corresponding cell cost.
Step – 4: Identify the minimum element in each row and column of the Transportation table and subtract in their corresponding the row and the column.
Step – 5: Find the sum of the values in the row and the column. Choose the maximum value and allocate the minimum of supply/demand in the minimum element of the row and column. Eliminate by deleting the columns or rows corresponding to where the supply or demand is satisfied.
Step – 6: Continue step -4 and step -5 until satisfaction of all the supply and demand is met.
Step – 7: Place the original transportation cost to satisfied cell cost.
Step – 8: Calculate the minimum cost.
That is,

$$\text{Total Cost} = \sum\sum C_{ij} X_{ij}$$

5. Numerical Example

Consider the Fuzzy Transportation Problem

	FD_1	**FD_2**	**FD_3**	**FD_4**	**Fuzzy Capacity**
FO_1	[1,2,3,4]	[1,3,4,6]	[9,11,12,14]	[5,7,8,11]	[1,6,7,12]
FO_2	[0,1,2,4]	[-1,0,1,2]	[5,6,7,8]	[0,1,2,3]	[0,1,2,3]
FO_3	[3,5,6,8]	[5,8,9,12]	[12,15,16,19]	[7,9,10,12]	[5,10,12,17]
Fuzzy Demand	[5,7,8,10]	[1,5,6,10]	[1,3,4,6]	[1,2,3,4]	

Solution

In Conformation to model the fuzzy transportation problem can be formulated in the following mathematical programming form

Min Z = R(1,2,3,4)x_{11} + R(1,3,4,6)x_{12} + R(9,11,12,14)x_{13} + R(5,7,8,11)x_{14} +R(0,1,2,4)x_{21}+R(-1,0,1,2)x_{22}+R(5,6,78)x_{23}+R(4,5,6,7)x_{24}+R(3,5,6,8)x_{31}+R(5,8,9,12)x_{32} +R(12,15,16,19)x_{33} + R(7,9,10,12)x_{34}

$$R(\tilde{a}) = \int_0^1 0.5(\mathrm{a}_\alpha^L \mathrm{a}_\alpha^U)\, d\alpha$$

Where $(\mathrm{a}_\alpha^L \mathrm{a}_\alpha^U) = \{((b-a)+a),(d-(d-c))\}$

After applying ranking technique, we get

Table -1

	D1	D2	D3	D4	Supply
O1	2.5	3.5	11.5	7.75	6.5
O2	1.75	0.5	6.5	1.5	1.5
O3	5.5	8.5	15.5	9.5	11
Demand	7.5	5.5	3.5	2.5	

Table – 2

	D1	D2	D3	D4	Supply
O1	2.5_{0}^{0}	3.5_{3}^{1}	11.5_{5}^{9}	$7.75_{6.25}^{5.25}$	6.5
O2	$1.75_{0.75}^{1.25}$	5_{0}^{0}	6.5_{0}^{6}	1.5_{0}^{1}	1.5
O3	5.5_{3}^{0}	8.5_{8}^{3}	15.5_{9}^{10}	9.5_{8}^{4}	11
Demand	7.5	5.5	3.5	2.5	

Table – 3

	D1	D2	D3	D4	Supply
O1	0	4	14	11.5	6.5
O2	2	0	6	1	1.5
O3	3	11	19	12	11
Demand	7.5	5.5	3.5	2.5	

With the help of the method, we get

Table – 4

	D1	D2	D3	D4	Supply
O1	0	5.5 4	1 14	11.5	6.5
O2	2	0	1.5 6	1	1.5
O3	7.5 3	11	1 19	2.5 12	11
Demand	7.5	5.5	3.5	2.5	

Finally, we get

Table – 5

	D1	**D2**	**D3**	**D4**	**Supply**
O1		**5.5** **3.5**	**1** **11.5**		6.5
O2			**1.5** **6.5**		1.5
O3	**7.5** **5.5**		**1** **15.5**	**2.5** **9.5**	11
Demand	7.5	5.5	3.5	2.5	

Hence (4+3-1)=6 cells are allocated and hence we got our feasible soln. Next we calculate total cost and its corresponding allocated value of supply and demand

Total Cost (5.5X7.5) + (3.5X5.5) + (11.5X1) + (6.5X1.5) + (15.5X1) + (9.5X2.5) = 121

This is a basic feasible solution. The solution obtained using NCM, LCM, VAM and MODI/Stepping stone methods respectively. Hence the basic feasible solution obtained from method is optional soln.

6. Conclusion

In this paper, the transportation costs are thought of as general numbers delineated by fuzzy numbers that square measure a lot of realistic and general in nature. Moreover, the fuzzy transportation problem of trapezoidal fuzzy numbers has been transformed into crisp transportation problem using robust ranking technique indices. Numerical examples show that by this technique we are able to have the optimum resolution also because the crisp and fuzzy optimum total value. By using robust ranking method, we have shown that the total cost obtained is optimal. Hence, this may be useful for call manufacturers United Nations agency square measure handling provision and provide chain issues in fuzzy setting. For future research, we propose effective implementation of the trapezoidal fuzzy numbers in all fuzzy problems.

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